

Quantum Computing CHEAT SHEET

for circuit magicians

Bits and Qubits

Instead of classical bits, quantum computers use **quantum bits** (or qubits for short).

Bit

- 0
- or
- 1

Qubit

Superposition
Linear combination between two or more states, e.g.
 $\sqrt{0.8}|0\rangle + \sqrt{0.2}e^{i\frac{\pi}{2}}|1\rangle$

Measurement
The full state is not accessible in one measurement, but multiple state preparations and measurements are needed to access the probability distribution.

80% 0
20% 1

Quantum states can also be described using vectors:
 $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

we call this a ket

One way to picture quantum states is the circle notation:

the inner circles represents the amplitude
the black line indicates the phase

$\sqrt{0.8}|0\rangle + \sqrt{0.2}e^{i\frac{\pi}{2}}|1\rangle$

Multiple qubits form a **register**. The number of computational states doubles with each new qubit. A state with multiple qubits involved is often denoted like $|00\rangle = |0\rangle \otimes |0\rangle$ (where \otimes is the tensor product)

# qubits	# basis states	example
1	2	$\frac{1}{\sqrt{2}} 0\rangle - \frac{1}{\sqrt{2}} 1\rangle$
2	4	$\frac{1}{2} 00\rangle + \frac{1}{2} 01\rangle + \frac{1}{2} 10\rangle + \frac{1}{2} 11\rangle$
3	8	$\frac{1}{2\sqrt{2}}(000\rangle - \frac{1}{2\sqrt{2}} 001\rangle - \frac{1}{2\sqrt{2}} 010\rangle + \frac{1}{2\sqrt{2}} 011\rangle - \frac{1}{2\sqrt{2}} 100\rangle + \frac{1}{2\sqrt{2}} 101\rangle - \frac{1}{2\sqrt{2}} 110\rangle + \frac{1}{2\sqrt{2}} 111\rangle)$

possible linear combination of two-qubit states

Two or more qubits can be **entangled**, meaning that the state cannot be factorized as a product of states:

$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle \neq (a|0\rangle + b|1\rangle)(c|0\rangle + d|1\rangle)$

Entanglement
Entanglement between two qubits can be created, for example, with this circuit

One-Qubit Gates

Gate	Matrix	Ket and circle notation
X Pauli-X is a 180° rotation around the x-axis; also known as the quantum NOT gate	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$a 0\rangle + b 1\rangle \xrightarrow{X} b 0\rangle + a 1\rangle$
Y Pauli-Y is a 180° rotation around the y-axis	$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$	$a 0\rangle + b 1\rangle \xrightarrow{Y} -ib 0\rangle + ia 1\rangle$
Z Pauli-Z is a 180° rotation around the z-axis	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$a 0\rangle + b 1\rangle \xrightarrow{Z} a 0\rangle - b 1\rangle$
H Hadamard maps $ 0\rangle$ to $ +\rangle$ and $ 1\rangle$ to $ -\rangle$; used to create an equal superposition	$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$ 0\rangle \xrightarrow{H} \frac{1}{\sqrt{2}}(0\rangle + 1\rangle)$ $ 1\rangle \xrightarrow{H} \frac{1}{\sqrt{2}}(0\rangle - 1\rangle)$
S S is a 90° rotation around the z-axis; $S^2 = Z$; The inverse S^\dagger rotates in the opposite direction	$\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$	$a 0\rangle + b 1\rangle \xrightarrow{S} a 0\rangle + be^{i\frac{\pi}{2}} 1\rangle$
T T is a 45° rotation around the z-axis; $T^2 = S$; The inverse T^\dagger rotates in the opposite direction	$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix}$	$a 0\rangle + b 1\rangle \xrightarrow{T} a 0\rangle + be^{i\frac{\pi}{4}} 1\rangle$

Quantum circuits are a model to visualize operations on qubits.

these boxes symbolize operations acting on one or multiple qubits and are called gates

Binary and decimal: You will find both the use of the binary representation of qubit states as well as the decimal representation.

Decimal	Binary
$ 0\rangle$	$ 000\rangle$
$ 1\rangle$	$ 001\rangle$
$ 2\rangle$	$ 010\rangle$
$ 3\rangle$	$ 011\rangle$
$ 4\rangle$	$ 100\rangle$
$ 5\rangle$	$ 101\rangle$
$ 6\rangle$	$ 110\rangle$
$ 7\rangle$	$ 111\rangle$

means that the first and second qubit are $|1\rangle$ and the third qubit is $|0\rangle$

Building Blocks for Quantum Algorithms

There are many clever ways to arrange quantum circuits. A couple of them are depicted below.

Increment & decrement are used to add or subtract one from a register and are an example of how to do arithmetic with quantum gates.

Swap test allows for checking how similar the states in two registers are.

this is often used to indicate that multiple qubits are represented with this line
1 if registers are in same state
this is a controlled SWAP gate (also called Fredkin gate)

Amplitude Amplification converts phase differences into amplitude differences. It can be used (multiple times) to increase the success probability of query or search algorithms like Grover's algorithm.

flip phase of target with appropriate circuit
CZ with two control qubits

Quantum Fourier Transform can reveal the signal frequency in a register. Among other algorithms, it is used in Shor's algorithm for factoring numbers and computing the discrete logarithm.

conditional phase gates with phase decreasing over the qubit count: $-90^\circ, -45^\circ, -22.5^\circ, \dots$

Multi-Qubit Gates

Gate	Matrix	Ket and circle notation
CNOT applies a Pauli-X gate to the target qubit if the state of the control qubit is $ 1\rangle$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	$a 00\rangle + b 01\rangle + c 10\rangle + d 11\rangle \xrightarrow{\text{CNOT}} a 00\rangle + b 01\rangle + d 10\rangle + c 11\rangle$
CZ applies a Pauli-Z gate to the target qubit if the state of the control qubit is $ 1\rangle$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$	$a 00\rangle + b 01\rangle + c 10\rangle + d 11\rangle \xrightarrow{\text{CZ}} a 00\rangle + b 01\rangle + c 10\rangle - d 11\rangle$
SWAP swaps the state of 2 qubits; can be implemented using 3 alternating CNOTs	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	$a 00\rangle + b 01\rangle + c 10\rangle + d 11\rangle \xrightarrow{\text{SWAP}} a 00\rangle + c 01\rangle + b 10\rangle + d 11\rangle$
Toffoli applies a Pauli-X gate to the target qubit if both control qubits are in state $ 1\rangle$; can be used to construct a reversible version of the classical AND-gate	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$a 000\rangle + b 001\rangle + c 010\rangle + d 011\rangle + e 100\rangle + f 101\rangle + g 110\rangle + h 111\rangle \xrightarrow{\text{Toffoli}} a 000\rangle + b 001\rangle + c 010\rangle + h 011\rangle + e 100\rangle + f 101\rangle + g 110\rangle + d 111\rangle$