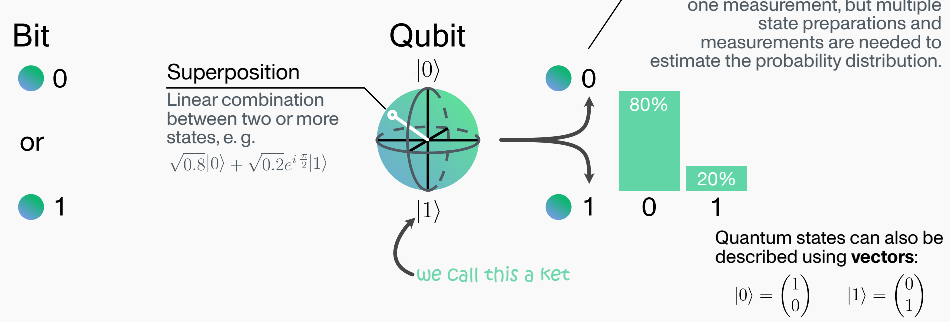
Quantum Computing CHEAT SHEET *for circuit magicians*

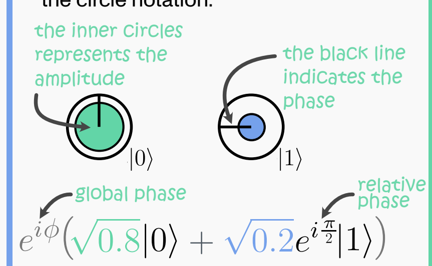
Bits and Qubits

Instead of classical bits, quantum computers use **quantum bits** (or qubits for short).



Quantum states can also be described using **vectors**: $|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ $|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$

One way to picture quantum states is the circle notation:



$$e^{i\phi}\left(\sqrt{0.8}|0\rangle + \sqrt{0.2}e^{i\frac{\pi}{4}}|1\rangle\right)$$

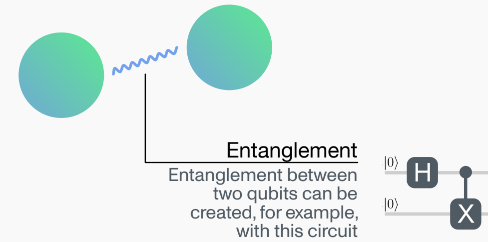
Only the phase of $|1\rangle$ relative to $|0\rangle$ is relevant for measurement. A global phase $e^{i\phi}$ has no physical meaning.

Multiple qubits form a **register**. The number of computational states doubles with each new qubit. A state with multiple qubits involved is often denoted like $|00\rangle = |0\rangle \otimes |0\rangle$ (where $\otimes$ is the tensor product)

# qubits	# basis states	example
1	2	$\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$
2	4	$\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle$ (possible linear combination of two-qubit states)
3	8	$\frac{1}{2\sqrt{2}}|000\rangle - \frac{1}{2\sqrt{2}}|001\rangle - \frac{1}{2\sqrt{2}}|010\rangle + \frac{1}{2\sqrt{2}}|011\rangle - \frac{1}{2\sqrt{2}}|100\rangle + \frac{1}{2\sqrt{2}}|101\rangle - \frac{1}{2\sqrt{2}}|110\rangle + \frac{1}{2\sqrt{2}}|111\rangle$

Two or more qubits can be **entangled**, meaning that the state cannot be factorized as a product of states:

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle \neq (a|0\rangle + b|1\rangle)(c|0\rangle + d|1\rangle)$$

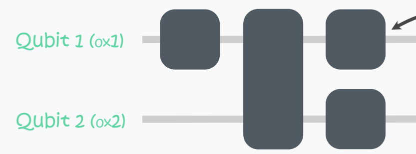


Entanglement
Entanglement between two qubits can be created, for example, with this circuit

One-Qubit Gates

Gate	Matrix	Ket and circle notation
X — **Pauli-X** is a 180° rotation around the x-axis; also known as the quantum NOT gate	$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$	$a|0\rangle + b|1\rangle$ → X → $b|0\rangle + a|1\rangle$
Y — **Pauli-Y** is a 180° rotation around the y-axis	$\begin{pmatrix}0 & -i\\i & 0\end{pmatrix}$	$a|0\rangle + b|1\rangle$ → Y → $-ib|0\rangle + ia|1\rangle$
Z — **Pauli-Z** is a 180° rotation around the z-axis	$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$	$a|0\rangle + b|1\rangle$ → Z → $a|0\rangle - b|1\rangle$
H — **Hadamard** maps $|0\rangle$ to $|+\rangle$ and $|1\rangle$ to $|-\rangle$; used to create an equal superposition	$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}$	$|0\rangle$ → H → $|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$; $|1\rangle$ → H → $|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$
S — **S** is a 90° rotation around the z-axis; S S = Z; The inverse S† rotates in the opposite direction	$\begin{pmatrix}1 & 0\\0 & i\end{pmatrix}$	$a|0\rangle + b|1\rangle$ → S → $a|0\rangle + be^{i\frac{\pi}{2}}|1\rangle$ ($e^{i\frac{\pi}{2}} = i$)
T — **T** is a 45° rotation around the z-axis; T T = S; The inverse T† rotates in the opposite direction	$\begin{pmatrix}1 & 0\\0 & e^{\frac{i\pi}{4}}\end{pmatrix}$	$a|0\rangle + b|1\rangle$ → T → $a|0\rangle + be^{i\frac{\pi}{4}}|1\rangle$

Quantum circuits are a model to visualize operations on qubits.



Binary and decimal: You will find both the use of the binary representation of qubit states as well as the decimal representation.

Decimal	Binary	Decimal	Binary
$|0\rangle$	$|000\rangle$	$|4\rangle$	$|100\rangle$
$|1\rangle$	$|001\rangle$	$|5\rangle$	$|101\rangle$
$|2\rangle$	$|010\rangle$	$|6\rangle$	$|110\rangle$
$|3\rangle$	$|011\rangle$	$|7\rangle$	$|111\rangle$

means that the first and second qubit are $|1\rangle$ and the third qubit is $|0\rangle$

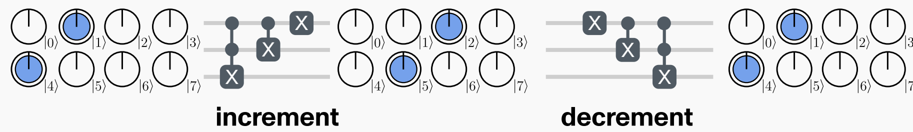
Multi-Qubit Gates

Gate	Matrix	Ket and circle notation
CNOT applies a Pauli-X gate to the target qubit if the state of the control qubit state is $|1\rangle$ (control qubit, target qubit)	$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$	$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$ → $a|00\rangle + b|01\rangle + d|10\rangle + c|11\rangle$
CZ applies a Pauli-Z gate to the target qubit if the state of the control qubit is $|1\rangle$	$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix}$	$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$ → $a|00\rangle + b|01\rangle + c|10\rangle - d|11\rangle$
SWAP swaps the state of 2 qubits; can be implemented using 3 alternating CNOTs	$\begin{pmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{pmatrix}$	$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$ → $a|00\rangle + c|01\rangle + b|10\rangle + d|11\rangle$
Toffoli applies a Pauli-X gate to the target qubit if both control qubits are in state $|1\rangle$; can be used to construct a reversible version of the classical AND-gate	$\begin{pmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\end{pmatrix}$	$a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle$ → $a|000\rangle + b|001\rangle + c|010\rangle + h|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + d|111\rangle$

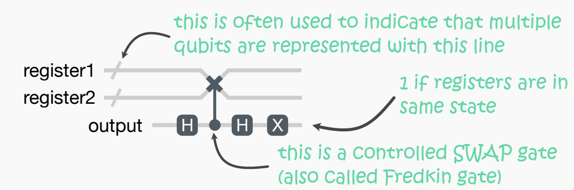
Building Blocks for Quantum Algorithms

There are many clever ways to arrange quantum circuits. A couple of them are depicted below.

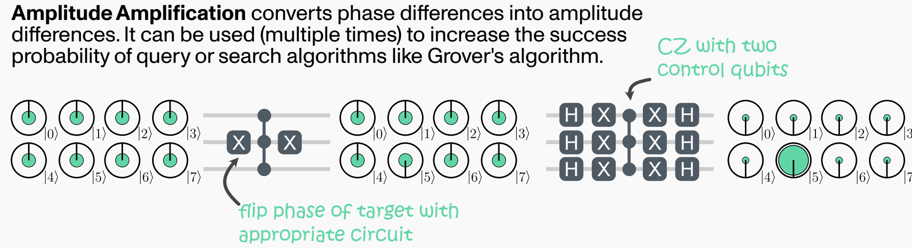
Increment & decrement are used to add or subtract one from a register and are an example of how to do arithmetic with quantum gates.



Swap test allows for checking how similar the states in two registers are.



Amplitude Amplification converts phase differences into amplitude differences. It can be used (multiple times) to increase the success probability of query or search algorithms like Grover's algorithm.



Quantum Fourier Transform can reveal the signal frequency in a register. Among other algorithms, it is used in Shor's algorithm for factoring numbers and computing the discrete logarithm.